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Curry–Yorke route to shearless attractors and coexistence of attractors in dissipative nontwist systems

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ABSTRACT

The routes to chaos play an important role in predictions about the transitions from regular to irregular behavior in nonlinear dynamical systems, such as electrical oscillators, chemical reactions, biomedical rhythms, and nonlinear wave coupling. Of special interest are dissipative systems obtained by adding a dissipation term in a given Hamiltonian system. If the latter satisfies the so-called twist property, the corresponding dissipative version can be called a “dissipative twist system.” Transitions to chaos in these systems are well established; for instance, the Curry–Yorke route describes the transition from a quasiperiodic attractor on torus to chaos passing by a chaotic banded attractor. In this paper, we study the transitions from an attractor on torus to chaotic motion in dissipative nontwist systems. We choose the dissipative standard nontwist map, which is a non-conservative version of the standard nontwist map. In our simulations, we observe the same transition to chaos that happens in twist systems, known as a soft one, where the quasiperiodic attractor becomes wrinkled and then chaotic through the Curry–Yorke route. By the Lyapunov exponent, we study the nature of the orbits for a different set of parameters, and we observe that quasiperiodic motion and periodic and chaotic behavior are possible in the system. We observe that they can coexist in the phase space, implying in multistability. The different coexistence scenarios were studied by the basin entropy and by the boundary basin entropy.

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